

Casimir force measurements at “large” separations: the thermal Casimir force and patch effects

Diego A. R. Dalvit
Theoretical Division
Los Alamos National Laboratory



Collaborators



Steve Lamoreaux (Yale)

Alex Sushkov (Harvard)

Woo-Joong Kim (Seattle)

Ryan Behunin (LANL)

Francesco Intravaia (LANL)

Paulo Maia Neto (Rio)

Serge Reynaud (Paris)

Ricardo Decca (Indianapolis)

Roberto Onofrio (Dartmouth)

- The thermal Casimir force
- Measurements at large separations: torsion pendulum
- Electrostatic calibrations in Casimir measurements
- Casimir force measurement between Ge plates
PRL **103**, 060401 (2009)
- Electrostatic patch effects
PRA **81**, 022505 (2010)
- Casimir force measurement between Au plates
Nature Physics **7**, 230 (2011)

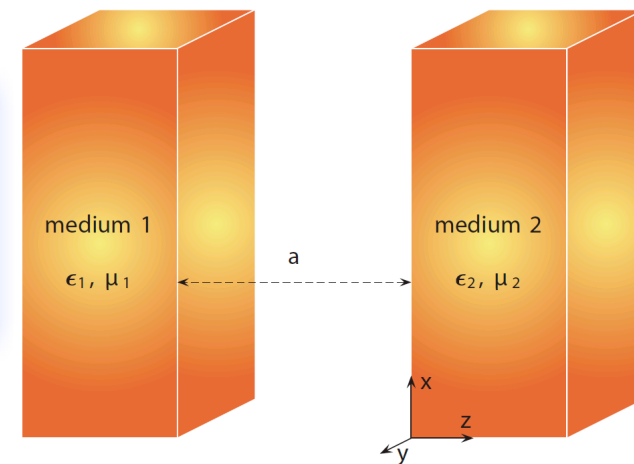
The thermal Casimir force

The Lifshitz formula

$T > 0$

$$\frac{F}{A} = \text{Im} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 \mathbf{k}_\parallel}{(2\pi)^2} \hbar \omega \coth \left(\frac{\hbar \omega}{2k_B T} \right) K_3 \text{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{2iK_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{2iK_3 d}}$$

$$K_3 = \sqrt{\omega^2/c^2 - k_\parallel^2}$$



$T = 0$

$$\frac{F}{A} = 2\hbar \text{Im} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 \mathbf{k}_\parallel}{(2\pi)^2} K_3 \text{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{2iK_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{2iK_3 d}}$$

Reflection matrices (Fresnel formulas for isotropic media):

$$r^{\text{TM}, \text{TM}}(\omega, \mathbf{k}_\parallel) = \frac{\epsilon(\omega) K_3 - \sqrt{\epsilon(\omega) \mu(\omega) \omega^2 / c^2 - k_\parallel^2}}{\epsilon(\omega) K_3 + \sqrt{\epsilon(\omega) \mu(\omega) \omega^2 / c^2 - k_\parallel^2}}$$

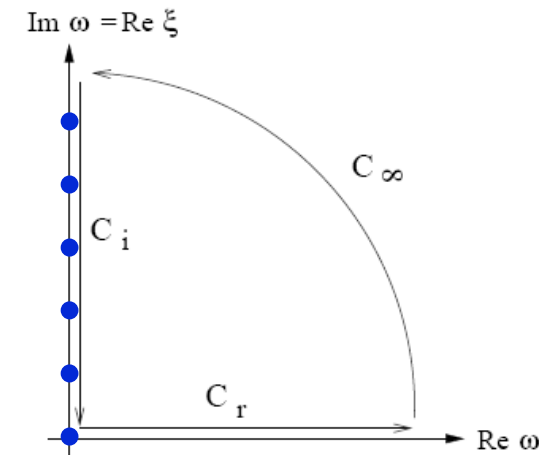
$$r^{\text{TE}, \text{TE}}(\omega, \mathbf{k}_\parallel) = \frac{\mu(\omega) K_3 - \sqrt{\epsilon(\omega) \mu(\omega) \omega^2 / c^2 - k_\parallel^2}}{\mu(\omega) K_3 + \sqrt{\epsilon(\omega) \mu(\omega) \omega^2 / c^2 - k_\parallel^2}}$$

Going to imaginary frequencies

The function $\coth(\hbar\omega/2k_B T)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m, \quad \xi_m = m \frac{2\pi k_B T}{\hbar}$$

After Wick rotation:



$$\frac{F}{A} = 2k_B T \sum_{m=0}^{\infty} \int \frac{d^2 \mathbf{k}_{\parallel}}{(2\pi)^2} K_3(i\xi_m) \text{Tr} \frac{\mathbf{R}_1(i\xi_m) \cdot \mathbf{R}_2(i\xi_m) e^{-2K_3(i\xi_m)d}}{1 - \mathbf{R}_1(i\xi_m) \cdot \mathbf{R}_2(i\xi_m) e^{-2K_3(i\xi_m)d}}$$

Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega$$

$$\mu(i\xi) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega \mu''(\omega)}{\omega^2 + \xi^2} d\omega$$

Casimir physics is a broad-band frequency phenomenon

The thermal “problem”

M. Boström and B.E. Sernelius,
Phys. Rev. Lett. **84** (2000) 4757

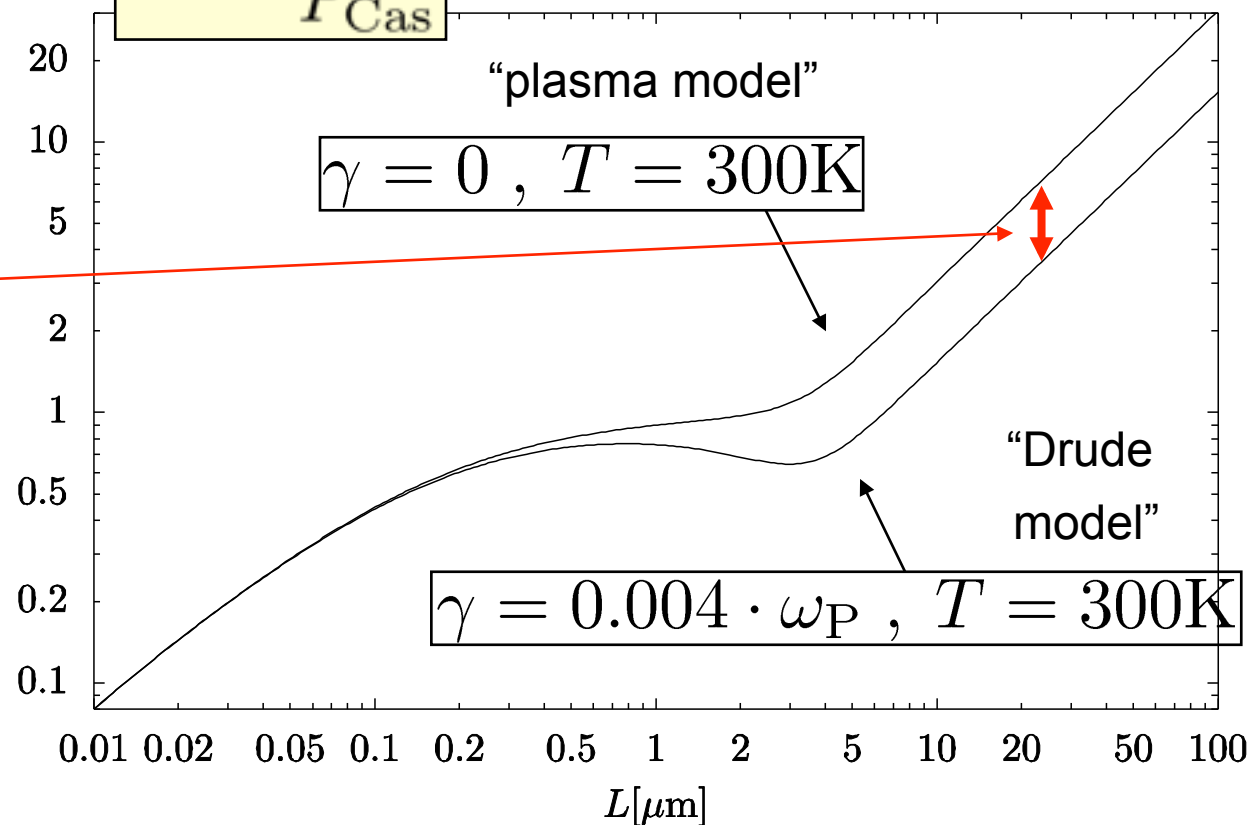
$$\eta_P = \frac{P}{P_{\text{Cas}}}$$

- Big effect of dissipation at large distances (factor 2)

Drawn here for
parameters of Gold

$$\lambda_P = 136 \text{ nm}$$

$$\gamma = 4 \times 10^{-3} \omega_P$$



Drude and plasma models

$$r^{\text{TM}} = \frac{\epsilon(i\xi) \sqrt{\xi^2/c^2 + k_{\parallel}^2} - \sqrt{\epsilon(i\xi)\xi^2/c^2 + k_{\parallel}^2}}{\epsilon(i\xi) \sqrt{\xi^2/c^2 + k_{\parallel}^2} + \sqrt{\epsilon(i\xi)\xi^2/c^2 + k_{\parallel}^2}}$$

$$r^{\text{TE}} = \frac{\sqrt{\xi^2/c^2 + k_{\parallel}^2} - \sqrt{\epsilon(i\xi)\xi^2/c^2 + k_{\parallel}^2}}{\sqrt{\xi^2/c^2 + k_{\parallel}^2} + \sqrt{\epsilon(i\xi)\xi^2/c^2 + k_{\parallel}^2}}$$

Drude model

$$\epsilon_D(i\xi) = 1 + \frac{\omega_P^2}{\xi(\xi + \gamma)}$$

Plasma model

$$\epsilon_P(i\xi) = 1 + \frac{\omega_P^2}{\xi^2}$$

At large separations, where thermal corrections are important, only the low-frequency behavior of the permittivity matters

$$\epsilon_D \propto 1/\xi$$

$$\lim_{\xi \rightarrow 0} r_D^{\text{TM}} = 1$$

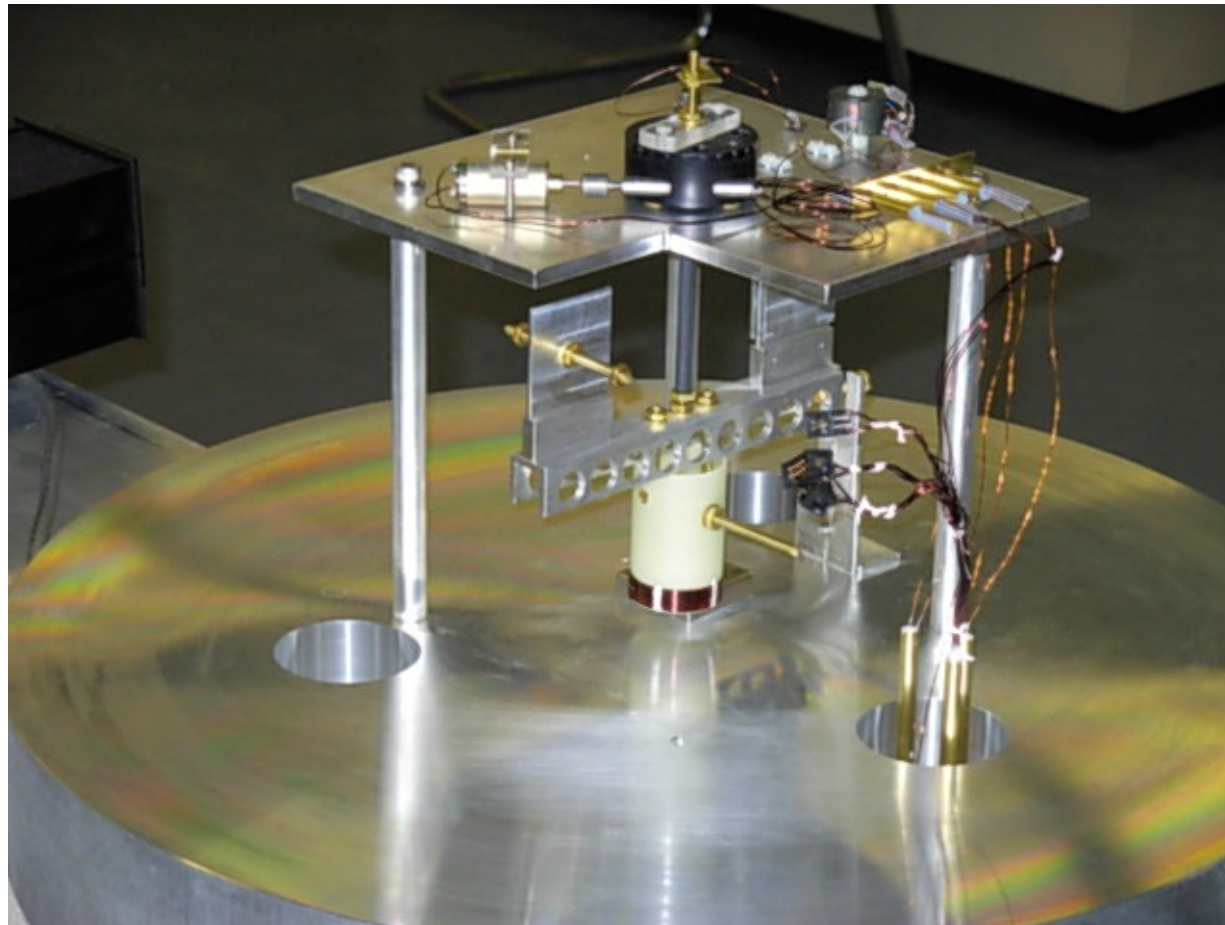
$$\lim_{\xi \rightarrow 0} r_D^{\text{TE}} = 0$$

$$\epsilon_P \propto 1/\xi^2$$

$$\lim_{\xi \rightarrow 0} r_P^{\text{TM}} = 1$$

$$\lim_{\xi \rightarrow 0} r_P^{\text{TE}} \neq 0$$

How to measure this?



Torsional pendulum

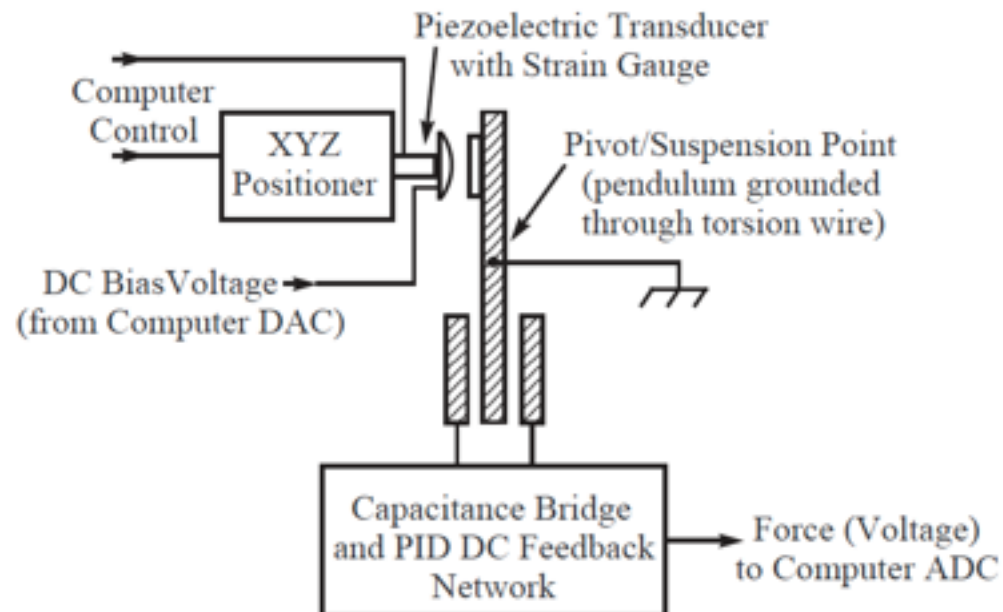
Experiment by Lamoreaux group (Yale)

● Sphere-plane geometry:

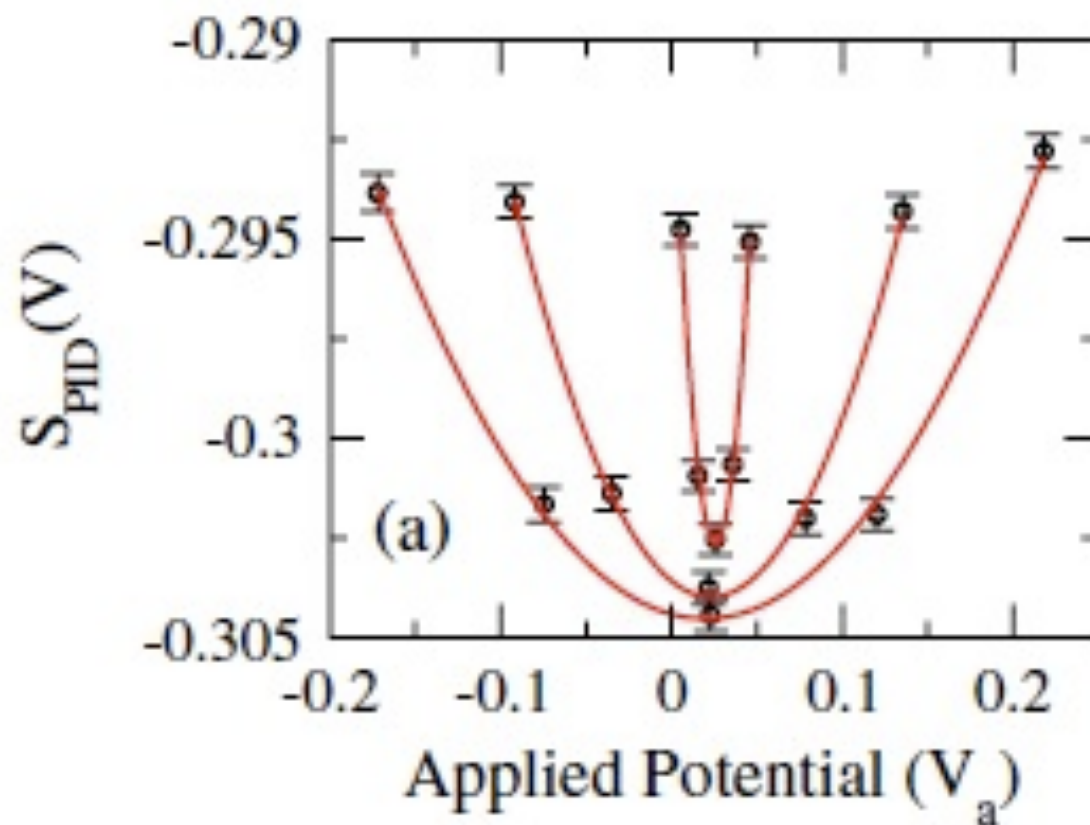
$$R = 15.1 \text{ cm}$$

● Torsional pendulum (modern Cavendish-like)

● Feedback control



Electrostatic calibrations



Typical Casimir measurement

$$S_{\text{PID}}(d, V_a) = S_{\text{dc}}(d \rightarrow \infty) + S_a(d, V_a) + S_r(d)$$

force-free component of
signal at large separations

electrostatic signal in
response to an applied
external voltage

residual signal due to
distance-dependent
forces, e.g. Casimir

The electrostatic signal between the spherical lens and the plate, in PFA ($d \ll R$) is

$$S_a(d, V_a) = \pi \epsilon_0 R (V_a - V_m)^2 / \beta d \quad \beta \text{ force-voltage conversion factor}$$

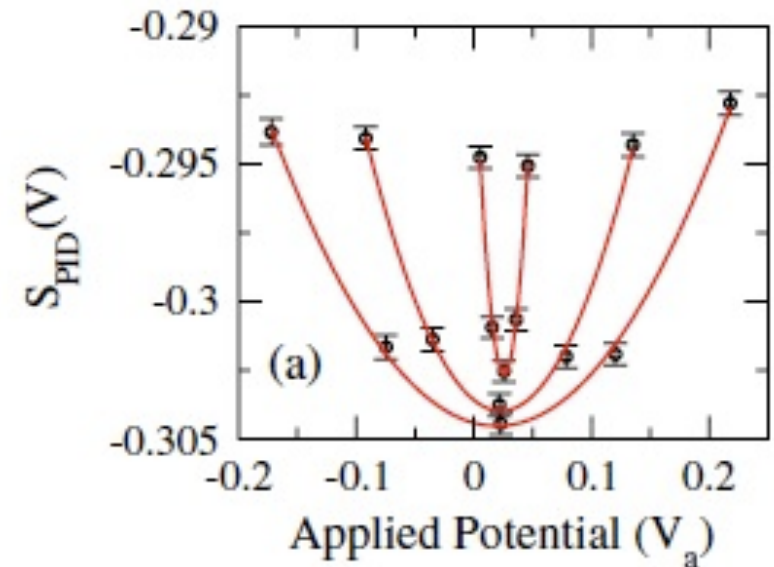
This signal is minimized ($S_a = 0$) when $V_a = V_m$, and the electrostatic minimizing potential V_m is then defined to be the **contact potential** between the plates.

“Parabola” measurements

Calibration routine

A range of plate voltages V_a is applied, and at a given nominal absolute distance the response is fitted to a parabola

$$S_{\text{PID}}(d, V_a) = S_0 + k(V_a - V_m)^2$$



Fitting parameters

- $k = k(d)$ \longrightarrow voltage-force calibration factor + absolute distance
- $V_m = V_m(d)$ \longrightarrow distance-dependent minimizing potential
- $S_0 = S_0(d)$ \longrightarrow force residuals: electrostatic + Casimir + non-Newtonian gravity +

Curvature parameter $k(d)$

From the curvature of the different parabolas one obtains $k(d)$

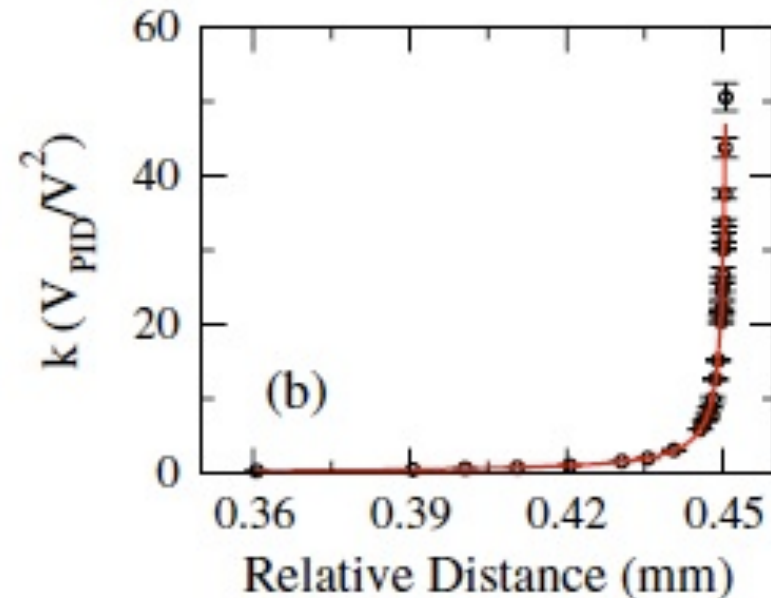
$$k(d) = \frac{\pi\epsilon_0 R/\beta}{d}$$

☑ Force-voltage calibration factor

$$\beta = (1.35 \pm 0.04) \times 10^{-7} \text{ N/V}$$

☑ Sphere-plane absolute distance

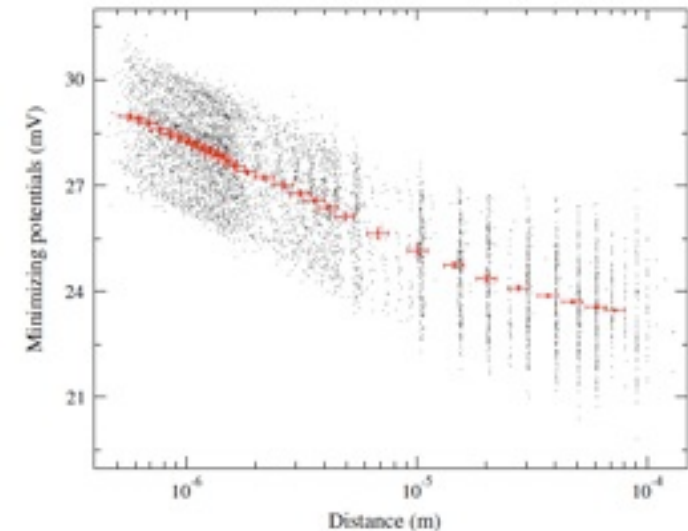
$$d = d_0 - d_{\text{rel}}$$



Minimizing potential

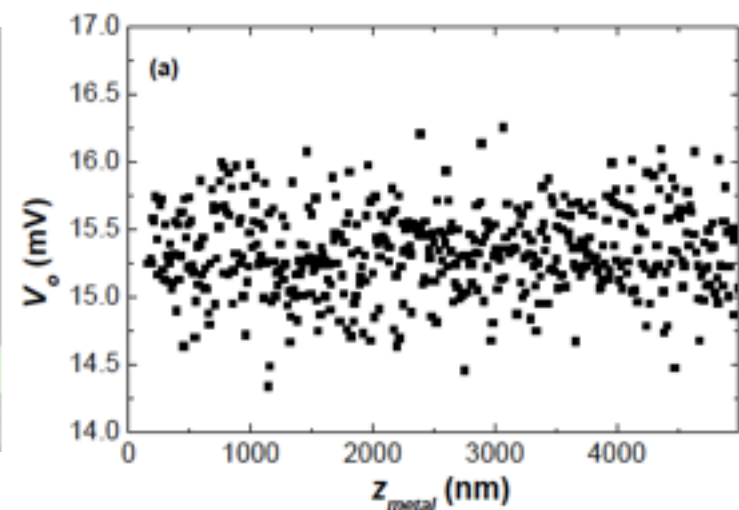
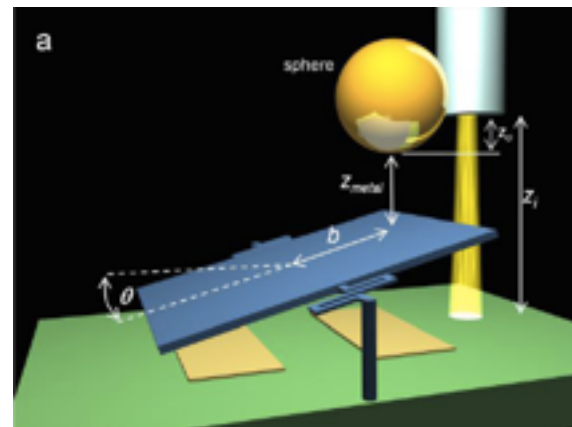
Our Ge data shows a *distance-dependent minimizing potential*, of the order of 6 mV over 100 μm .

$$V_m = V_m(d)$$



However, in some other experiments, the minimizing potential is *distance-independent*

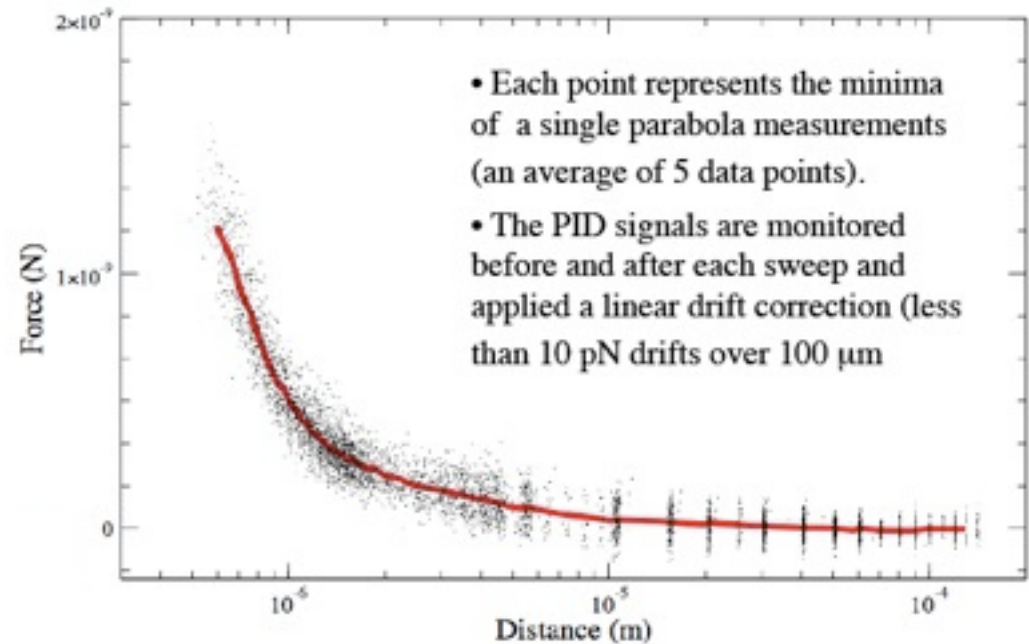
E.g.: Decca group



Force residuals in Ge experiment

Residuals from Coulomb force obtained from the value of the PID signal at the minima of each parabola,

$$S_0(d) \rightarrow F_r(d)$$



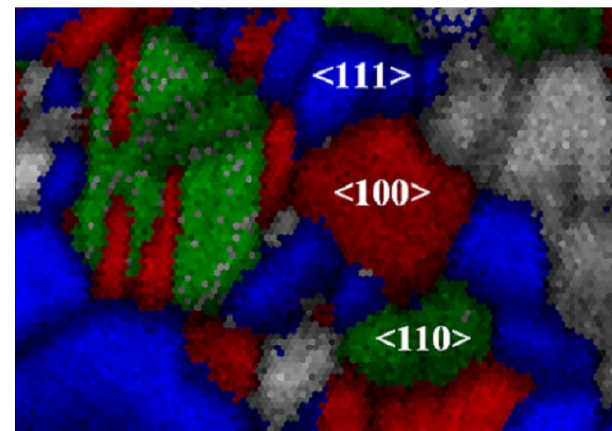
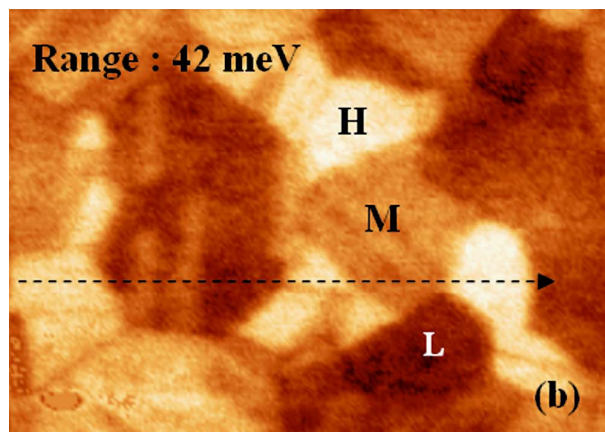
In our experiment, these force residuals are too large to be explained just by the Casimir-Lifshitz force between the Ge plates.

In fact, the experimental data shows a $1/d$ force residual at distances $d > 5\mu\text{m}$, where the Casimir force should be negligible.

What is the origin of the varying minimizing potential?

What is the origin of the additional force residual?

Electrostatic patches



Metals are NOT equipotentials

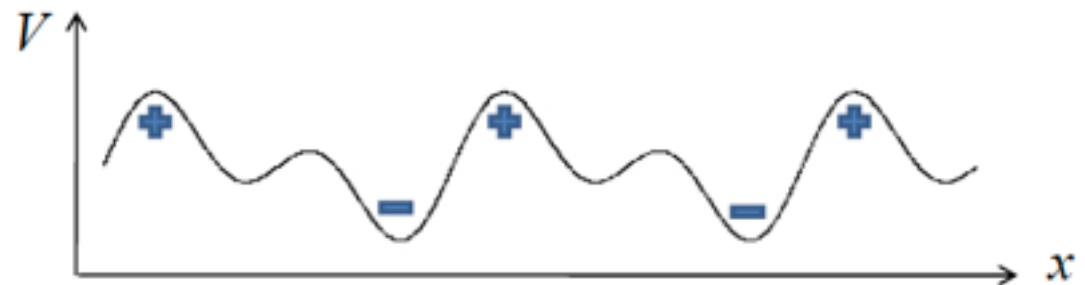
- Despite what we have learned in freshman physics!

- Different crystal faces have different work functions

Au crystal direction	Work function
$\langle 100 \rangle$	5.47 eV
$\langle 110 \rangle$	5.37 eV
$\langle 111 \rangle$	5.31 eV

- Dirt: oxides, surface adsorbates strongly affect work function and surface potential by creating dipoles on the surface.

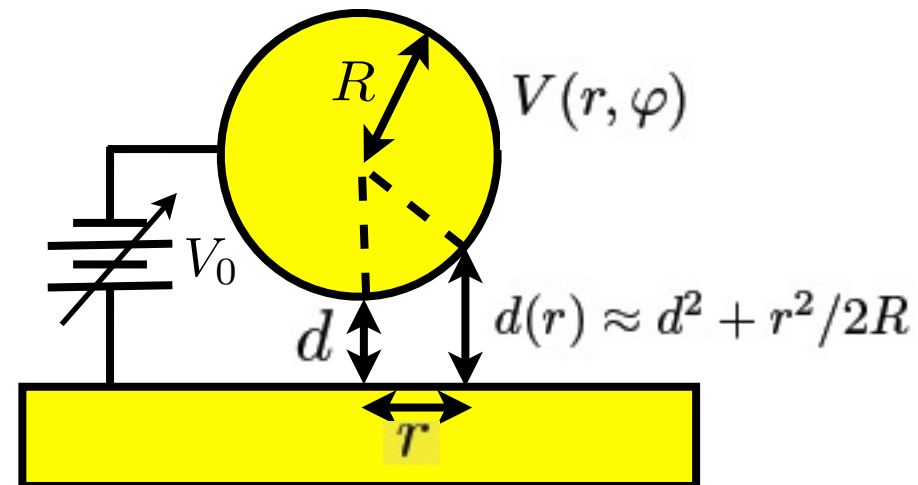
Resulting potential variation across a surface:



Surface potentials & $V_m(d)$

Electrostatic force (in PFA, $R \gg d$):

$$F(d, V_0) = \frac{\epsilon_0}{2} \int_0^{2\pi} d\varphi \int_0^R r dr \frac{(V(r, \varphi) + V_0)^2}{(d + r^2/2R)^2}$$



Minimized force at a fixed distance determines the minimizing potential $V_m(d)$

$$0 = \left. \frac{\partial F(d, V_0)}{\partial V_0} \right|_{V_0=V_m} = \epsilon_0 \int_0^{2\pi} d\varphi \int_0^R r dr \frac{V(r, \varphi) + V_m}{(d + r^2/2R)^2}$$

$$\Rightarrow V_m = V_m(d)$$

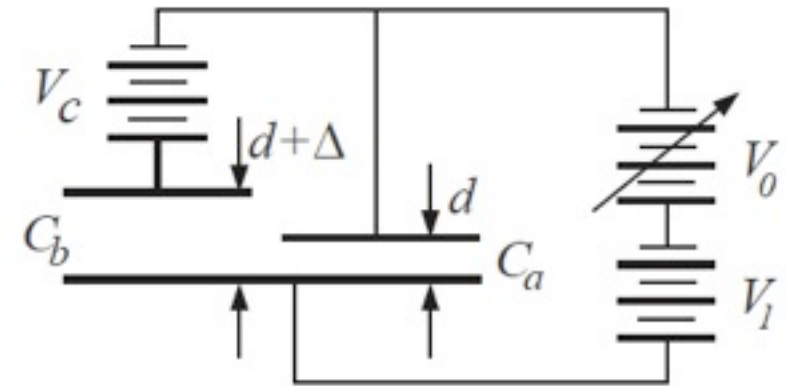
A toy model

A toy model illustrating the mechanism for the generation of $V_m(d)$ and $F_{\text{res}}^{\text{el}}(d)$

Force on lower plate:

$$F(d, V_0) = -\frac{1}{2}C'_a V_0^2 - \frac{1}{2}C'_b (V_0 + V_c)^2$$

(V_0 is varied, V_c a fixed property of the plates)



$$C'_a = -\epsilon_0 A / d^2$$

$$C'_b = -\epsilon_0 A / (d + \Delta)^2$$

When force is minimized, one gets a varying minimizing potential and a varying electrostatic residual force.

$$\left. \frac{\partial F(d, V_0)}{\partial V_0} \right|_{V_0=V_m} = 0 \Rightarrow V_m(d) = -\frac{C'_b V_c}{C'_a + C'_b} = -V_c \frac{d^2}{d^2 + (d + \Delta)^2}$$

$$F_{\text{res}}^{\text{el}}(d) = F(d, V_0 = V_m(d)) = \frac{\epsilon_0 A}{2} \frac{V_m^2(d) [d^2 + (d + \Delta)^2]}{d^4} \propto \frac{1}{d^4} \text{ for } \Delta \gg d$$

In reality, measurements can determine $V_m(d)$ up to an overall constant: $V_m(d) \rightarrow V_m(d) + V_1$

Electrostatic force residual

Sphere-plane case: $C'_a(d) = -2\pi\epsilon_0 R/d$

Dividing the sphere into infinitesimal areas, each with a random potential, and integrating over the surface to get the net residual force (as in PFA), we get

$$F_{\text{res}}^{\text{el}}(d) = \pi\epsilon_0 R \frac{[V_m(d) + V_1]^2}{d}$$

Important message from this analysis:

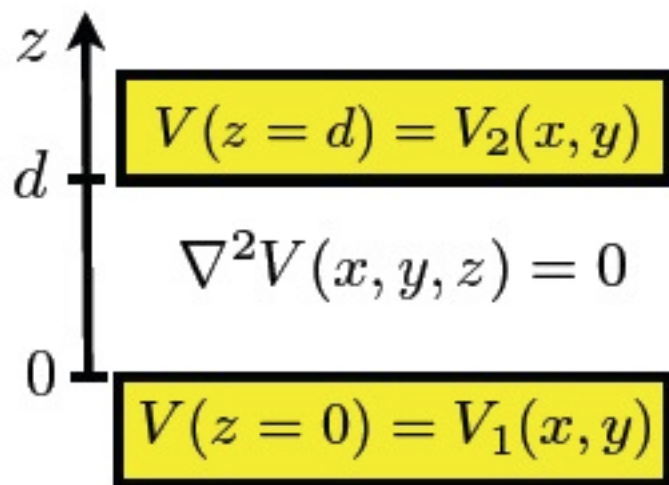


Minima of parabolas DO NOT nullify all possible electrostatic forces between plates!

Modeling patches

The patch effect is a possible systematic limitation to Casimir force measurements (Speake and Trenkel, PRL 03).

Plane-plane geometry:



$$V(x, y, z) = X(x)Y(y)Z(z)$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -\alpha^2; \quad \frac{1}{Y} \frac{d^2 Y}{dy^2} = -\beta^2; \quad \frac{1}{Z} \frac{d^2 Z}{dz^2} = \gamma^2 = \alpha^2 + \beta^2$$

$$V_1(x, y) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} V_{1,\mathbf{k}} \cos(k_x x) \cos(k_y y) \quad [\text{idem for } V_2(x, y)]$$

$$V(x, y, z) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{\cos(k_x x) \cos(k_y y)}{2 \sinh(\gamma d)} \times [e^{\gamma z} (V_{2,\mathbf{k}} - V_{1,\mathbf{k}} e^{-\gamma d}) + e^{-\gamma z} (V_{1,\mathbf{k}} e^{\gamma d} - V_{2,\mathbf{k}})]$$

Electrostatic energy:

$$U_{pp}(d) = \frac{\epsilon_0}{2} \int d^3 \mathbf{r} |\nabla V|^2$$

Random patches

Statistical properties for patch potentials:

$$\langle V_{1,k} \rangle = \langle V_{2,k} \rangle = \langle V_{2,k} V_{1,k'} \rangle = 0;$$

$$\langle V_{1,k} V_{1,k'} \rangle = C_{1,k} \delta^2(\mathbf{k} - \mathbf{k}');$$

$$\langle V_{2,k} V_{2,k'} \rangle = C_{2,k} \delta^2(\mathbf{k} - \mathbf{k}'),$$

Averaging the interaction energy over different realizations of the stochastic patches, we get

$$\langle U_{pp} \rangle = \frac{\epsilon}{16} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{\gamma \sinh(2\gamma d)}{\sinh^2(\gamma d)} [C_{1,k} + C_{2,k}]$$

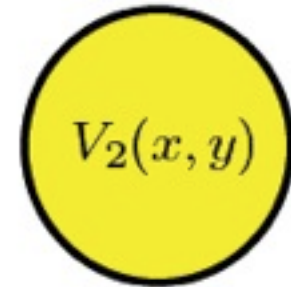
In the limit of large distances ($kd \gg 1$), this expression has an asymptotic behavior independent of distance (self-energy of each plate). We remove the potential energy at infinite separation, to get the **electrostatic interaction energy due to patch effects**

$$\langle U_{pp} \rangle = \frac{\epsilon_0}{32\pi} \int_0^\infty dk \frac{k^2 e^{-kd}}{\sinh(kd)} [C_{1,k} + C_{2,k}]$$

Patch force in sphere-plane

Sphere-plane geometry:

To compute the patch effect in the sphere-plane configuration we use PFA for the curvature effect ($d \ll R$) but leave kd arbitrary



$$\nabla^2 V(x, y, z) = 0$$

$$F_{sp}(d) = 2\pi R \langle U_{pp}(d) \rangle = \frac{\epsilon_0 R}{16} \int_0^\infty dk \frac{k^2 e^{-kd}}{\sinh(kd)} [C_{1,k} + C_{2,k}]$$

$$V(z=0) = V_1(x, y)$$

Different models to describe surface potential fluctuations:

🕒 $C_{1,k} = C_{2,k} = V_0^2$ for $k_{\min} < k < k_{\max}$

$$F_{sp} = \frac{4\pi\epsilon_0 V_{\text{rms}}^2 R}{k_{\max}^2 - k_{\min}^2} \int_{k_{\min}}^{k_{\max}} dk \frac{k^2 e^{-kd}}{\sinh(kd)}$$

🕒 $\mathcal{R}(r) = \begin{cases} V_0^2 & \text{for } r \leq \lambda, \\ 0 & \text{for } r > \lambda. \end{cases}$

$$F_{sp} = 2\pi\epsilon_0 R \int_0^\infty du \, u \frac{J_1(u)}{e^{2ud/\lambda} - 1}$$

(Speake and Trenkel, PRL 03).

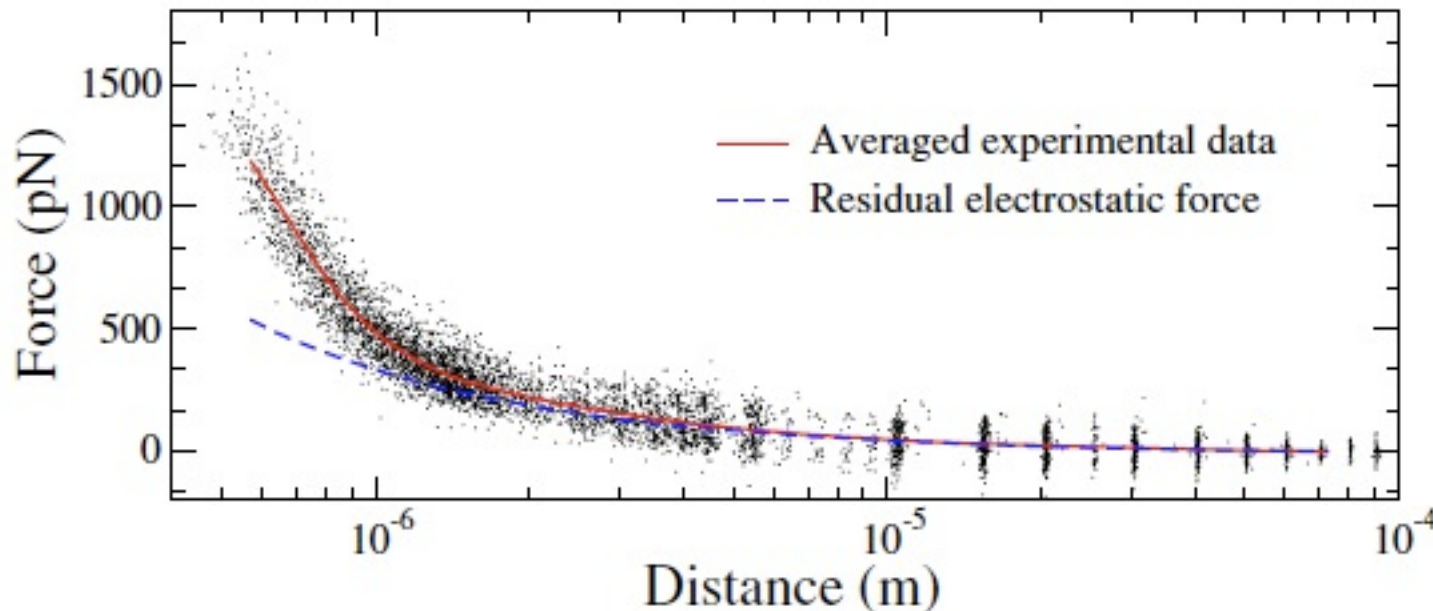
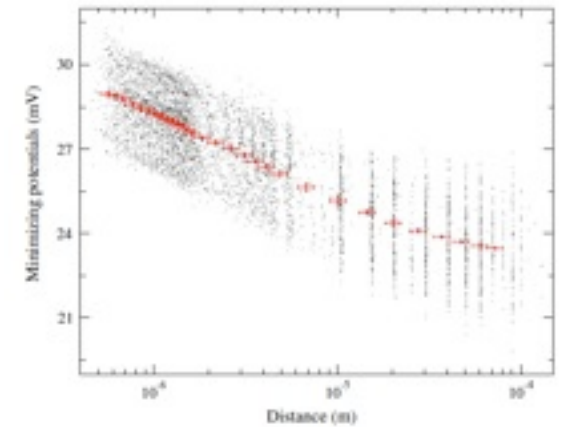
In the limit of large patches ($kd \ll 1$):

$$F_{sp}(d) = \pi\epsilon_0 R \frac{V_{\text{rms}}^2}{d}$$

Ge exp: patch fit at large distance

We fit the data for the *residual force at the minimizing potential* with a force of electric origin, for distances $d > 5\mu\text{m}$ (negligible Casimir)

$$F_r^{\text{el}}(d) = F_0 + \pi\epsilon_0 R \frac{[V_m(d) + V_1]^2 + V_{\text{rms}}^2}{d}$$



$$F_0 = (-11 \pm 2) \times 10^{-12} \text{ N}$$

$$V_1 = (-34 \pm 3) \text{ mV}$$

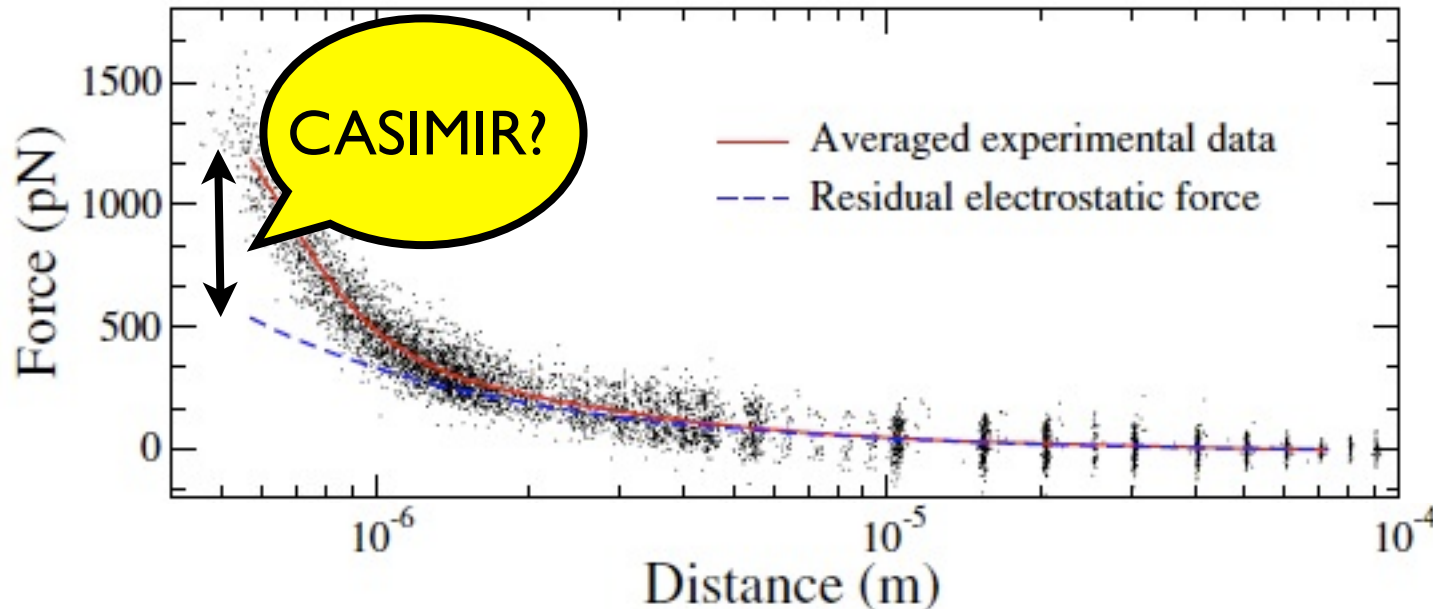
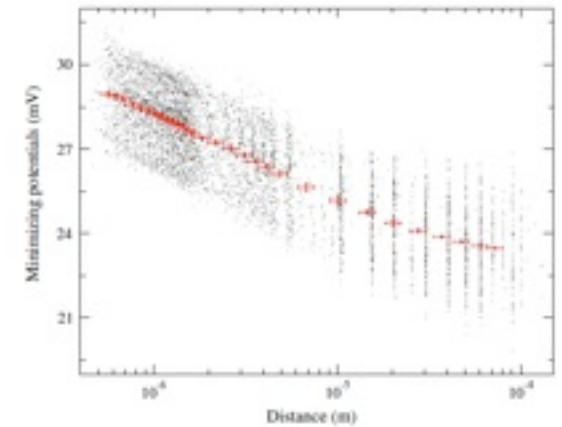
$$V_{\text{rms}} = (6 \pm 2) \text{ mV}$$

$$\chi_0^2 = 1.5$$

Ge exp: patch fit at large distance

We fit the data for the *residual force at the minimizing potential* with a force of electric origin, for distances $d > 5\mu\text{m}$ (negligible Casimir)

$$F_r^{\text{el}}(d) = F_0 + \pi\epsilon_0 R \frac{[V_m(d) + V_1]^2 + V_{\text{rms}}^2}{d}$$



$$F_0 = (-11 \pm 2) \times 10^{-12} \text{ N}$$

$$V_1 = (-34 \pm 3) \text{ mV}$$

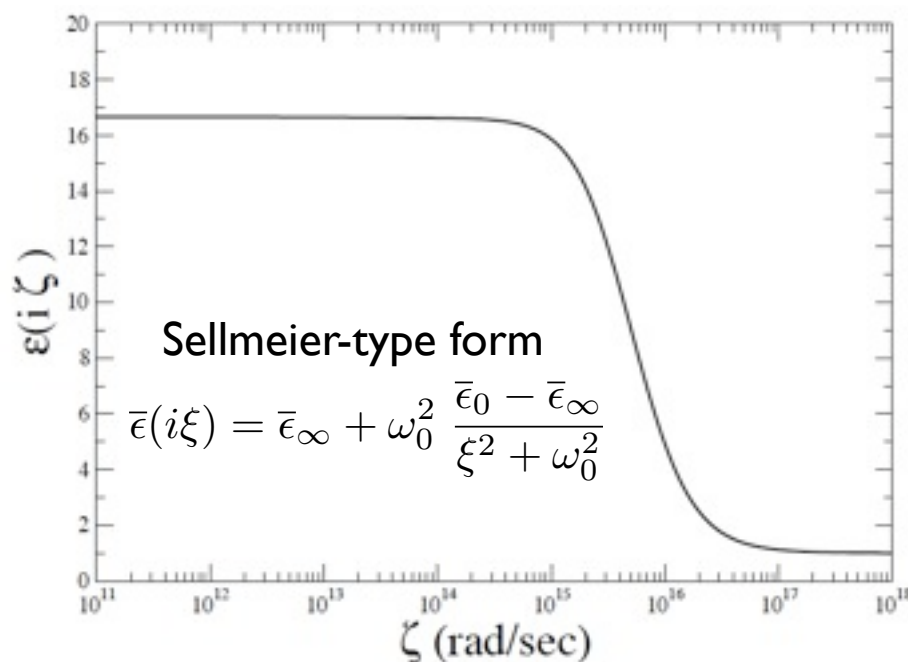
$$V_{\text{rms}} = (6 \pm 2) \text{ mV}$$

$$\chi_0^2 = 1.5$$

Material properties of Ge

- intrinsic semiconductor, among the purest materials available
- small density of free carriers (electrons and holes)
- conductivity, thermal, and optical properties are well tabulated

Bare permittivity of intrinsic Ge
(not including contributions from free carriers)



Ge reflection amplitudes

We need to compute the reflection amplitudes $r_{\mathbf{k},j}^p(\omega)$ for a vacuum-Ge interphase. Depending on the model used to describe the optical and conductivity properties of Ge we get different reflection amplitudes.

*** Ideal dielectric model:** No contribution from free carriers. Only the bare permittivity is taken into account. Reflection amplitudes are the usual Fresnel coefficients.

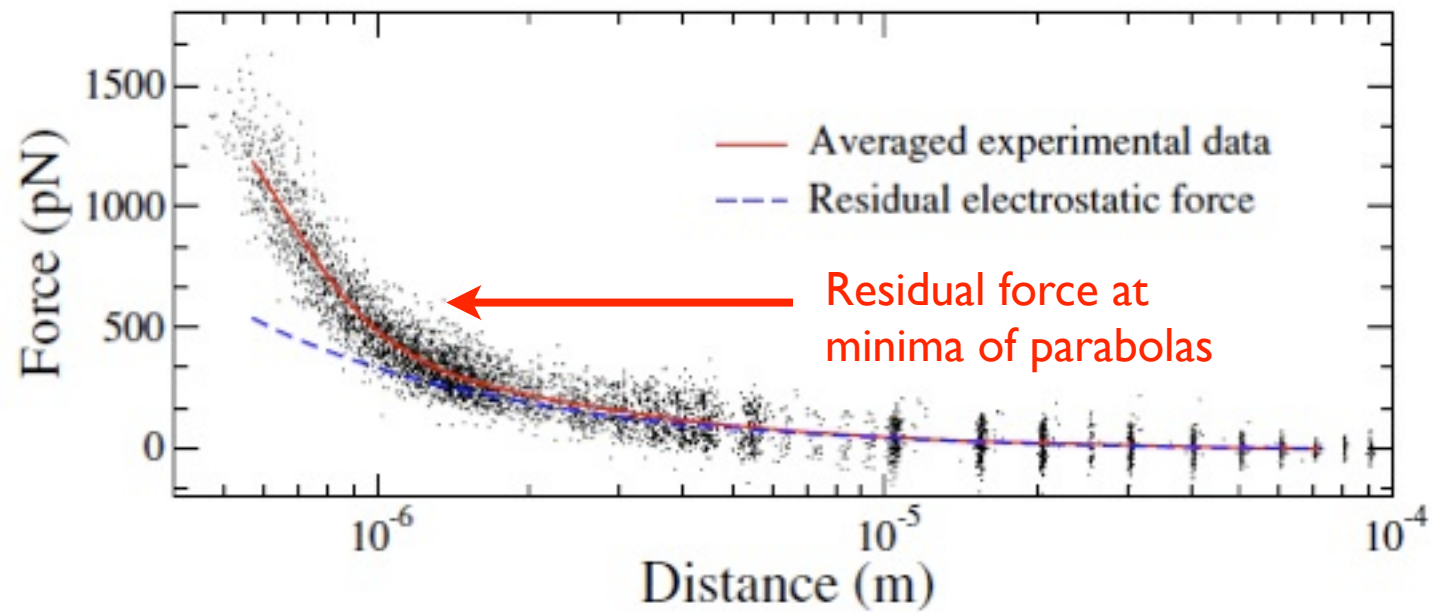
$$r_{\mathbf{k}}^{\text{TM}}(i\xi) = \frac{\sqrt{k^2 + \bar{\epsilon}(i\xi)\xi^2/c^2} - \bar{\epsilon}(i\xi)\sqrt{k^2 + \xi^2/c^2}}{\sqrt{k^2 + \bar{\epsilon}(i\xi)\xi^2/c^2} + \bar{\epsilon}(i\xi)\sqrt{k^2 + \xi^2/c^2}} \quad r_{\mathbf{k}}^{\text{TE}}(i\xi) = \frac{\sqrt{k^2 + \bar{\epsilon}(i\xi)\xi^2/c^2} - \sqrt{k^2 + \xi^2/c^2}}{\sqrt{k^2 + \bar{\epsilon}(i\xi)\xi^2/c^2} + \sqrt{k^2 + \xi^2/c^2}}$$

*** Ideal dielectric + Drude conductivity model:** An ac Drude conductivity term is added to the bare permittivity.

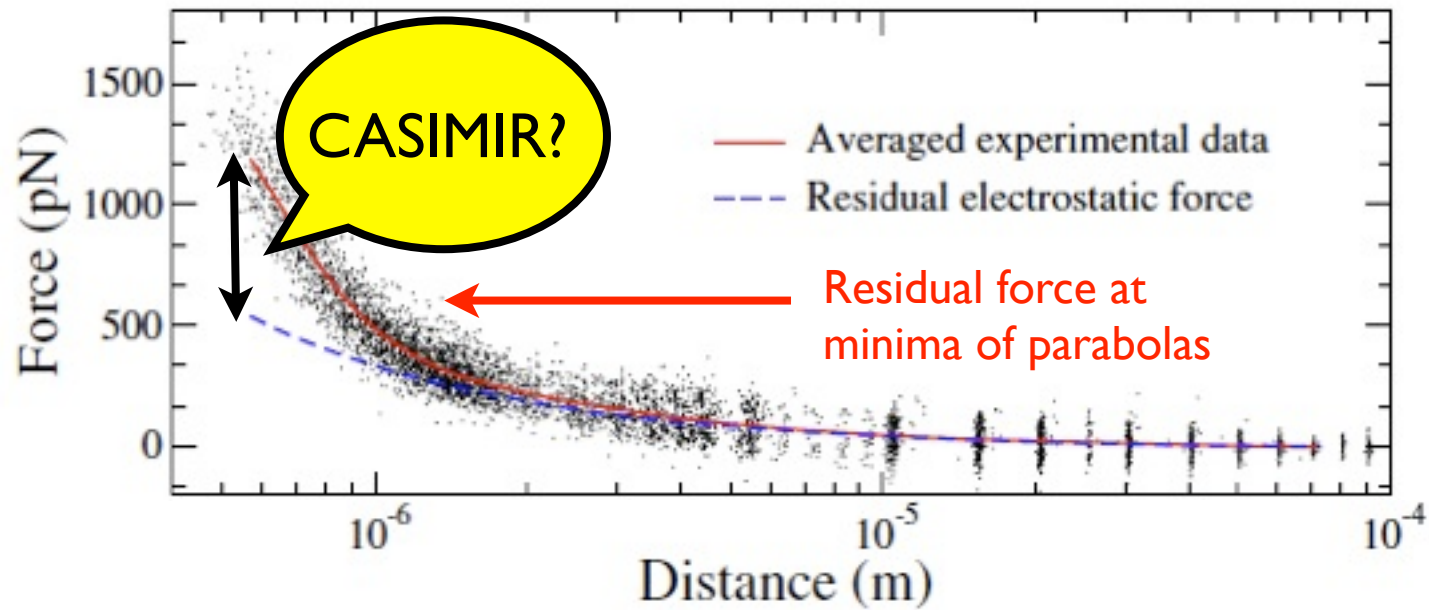
$$\epsilon(i\xi) = \bar{\epsilon}(i\xi) + \frac{4\pi\sigma(i\xi)}{\xi} \quad \sigma(i\xi) = \sigma_0/(1 + \xi\tau)$$
$$\sigma_0 = e^2 n_0 \tau / m_e \approx 1/(43 \, \Omega \, \text{cm})$$

Same Fresnel coefficients with the substitution $\bar{\epsilon}(i\xi) \rightarrow \epsilon(i\xi)$

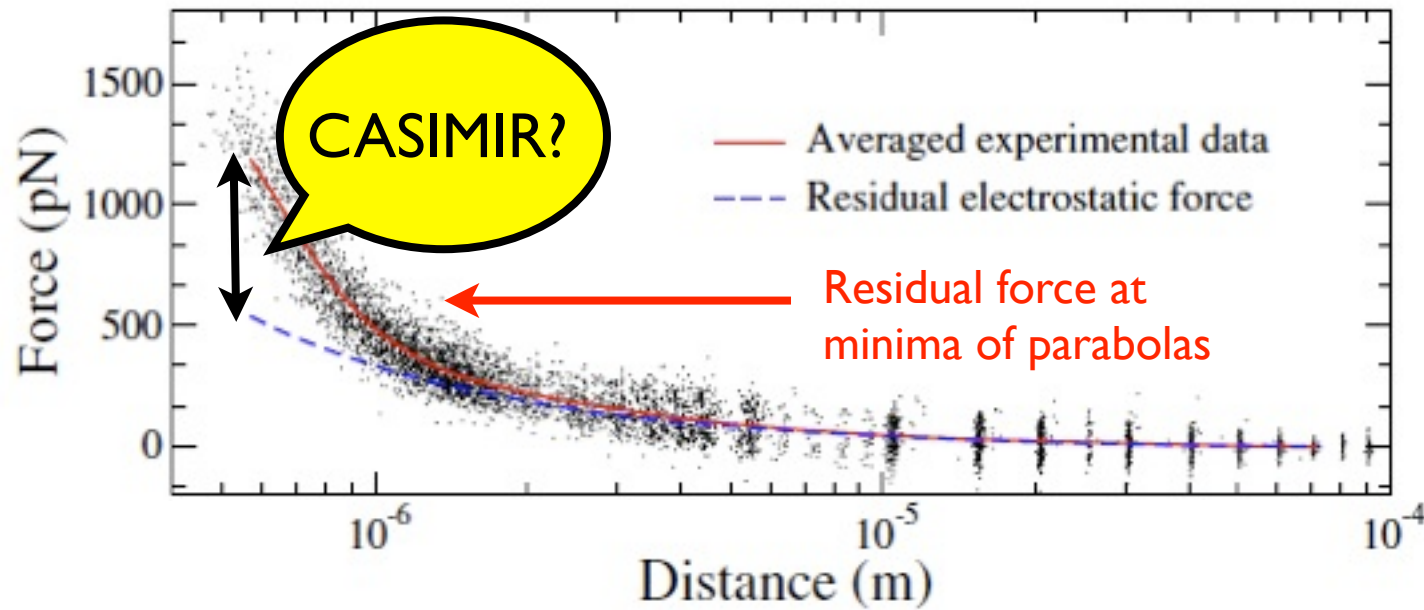
Ge experiment: patches+Casimir



Ge experiment: patches+Casimir

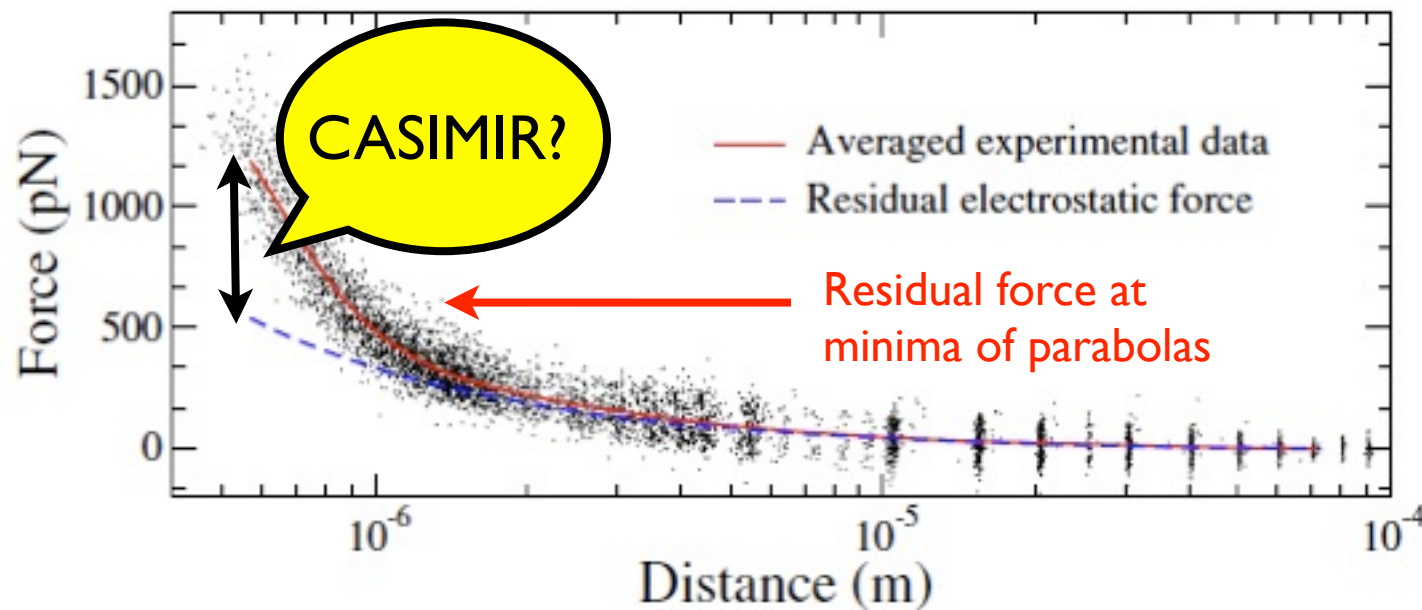


Ge experiment: patches+Casimir

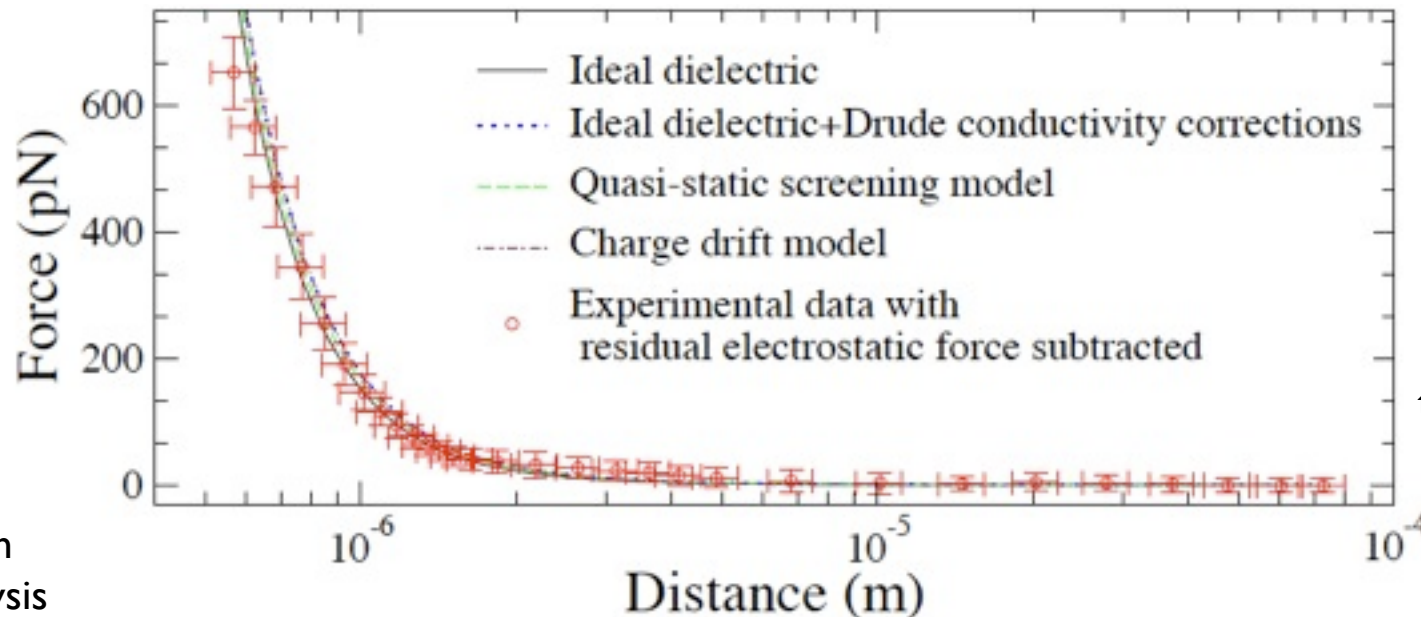


After subtraction of the electrostatic force residual $F_r^{\text{el}}(d) = F_0 + \pi\epsilon_0 R \frac{[V_m(d) + V_1]^2 + V_{\text{rms}}^2}{d}$

Ge experiment: patches+Casimir



After subtraction of the electrostatic force residual $F_r^{\text{el}}(d) = F_0 + \pi\epsilon_0 R \frac{[V_m(d) + V_1]^2 + V_{\text{rms}}^2}{d}$



Error bars:

3% statistical
uncertainties

10% fitting
uncertainties from
electrostatic analysis

For $d < 5\mu\text{m}$

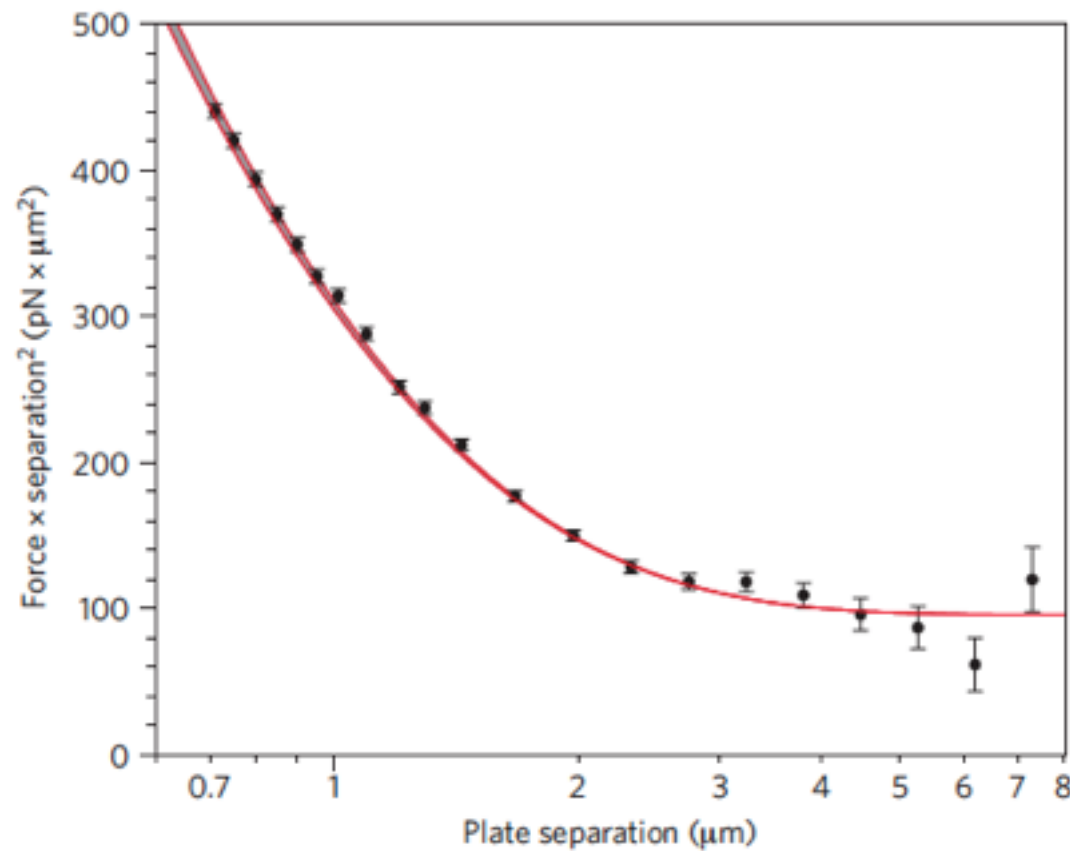
$$\chi_0^2 \approx 1$$

for all the
theoretical models

Remarks on the Ge experiment

- Found a distance-dependent minimizing potential, due to large-scale variations in the contact potential along the surface of the plates. It results in a relatively large residual force of electrostatic origin $\propto [V_m(d) + V_1]^2/d$
- Found another residual force of electrostatic origin, probably due to potential patches on the surfaces that, for $d \ll \lambda \ll R$, is $\propto V_{\text{rms}}^2/d$
- After subtraction of these two electrostatic residuals, we got very good agreement with a Casimir force residual. However, we do not have enough accuracy to distinguish between the different theoretical models.

Casimir force with Au plates



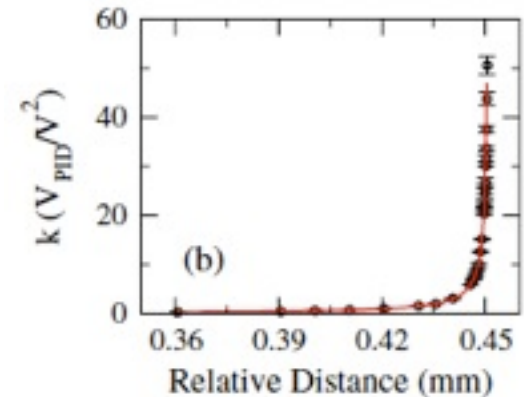
$k(d)$, $V_m(d)$, and $S_0(d)$

- From the parabola curvature one obtains the absolute distance

$$k(d) = \frac{\pi\epsilon_0 R/\beta}{d}$$

$$\beta = (1.27 \pm 0.04) \times 10^{-7} \text{ N/V}$$

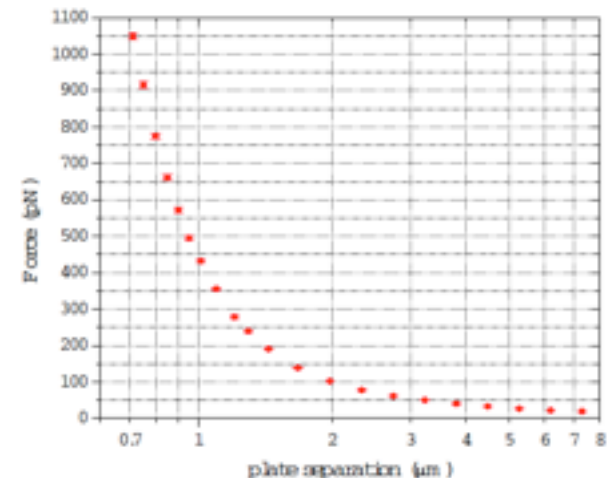
$$d = d_0 - d_{\text{rel}}$$



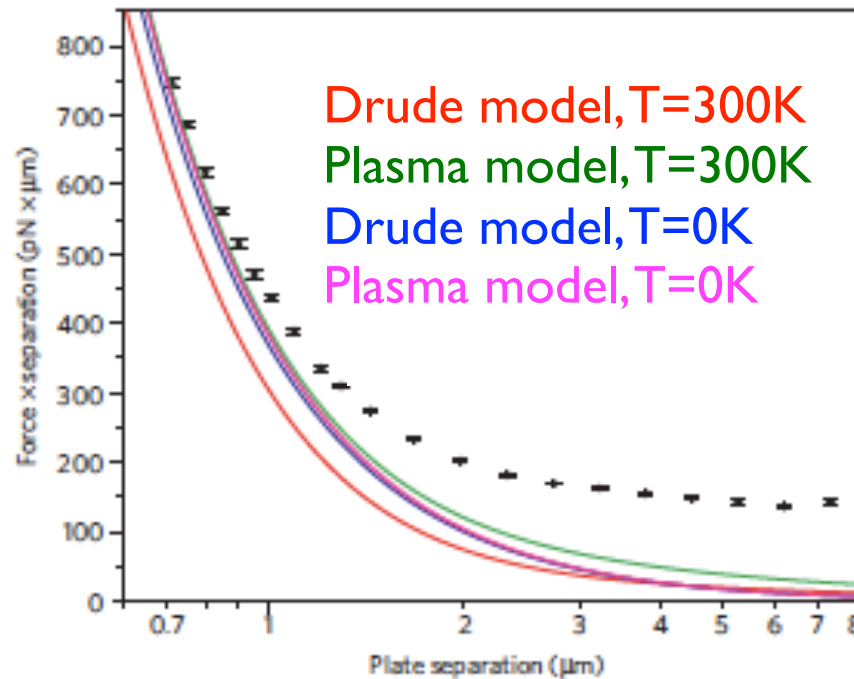
- From the parabola minimum one obtains the minimizing potential

Our Au data shows a *distance-independent minimizing potential* $V_m \approx 20 \text{ mV}$, with variations of 0.2 mV in the 0.7-7.0 μm range.

- From $S_0(d)$ one obtains the residual force $F_r(d)$



Au experiment: force residuals



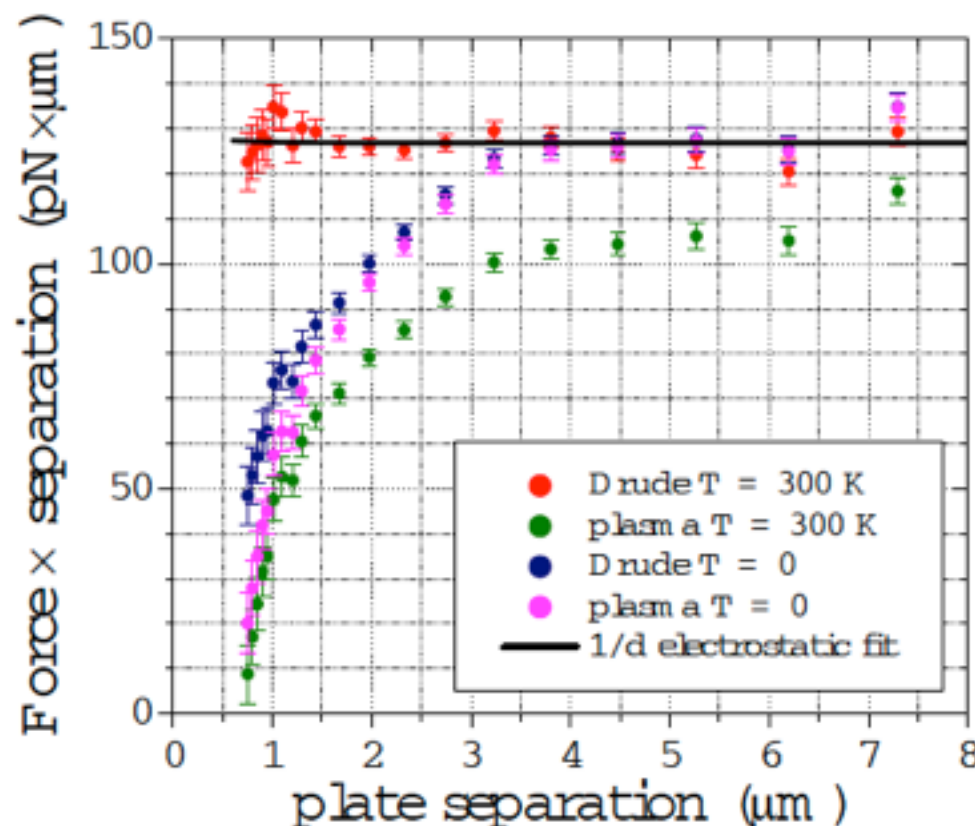
Solid lines correspond to predictions from Lifshitz theory (with no roughness correction) and Drude-like permittivity with parameters

$$\omega_p = 7.54 \text{ eV} \quad \gamma = 0.051 \text{ eV} \quad (\text{best fit to Au optical data by Palik})$$

In our experiment, these force residuals are too large to be explained just by the Casimir-Lifshitz force between Au plates.

Extracting the patch force

$$F_r - F_{\text{Casimir}} = \pi\epsilon_0 R V_{\text{rms}}^2 / d$$



Drude $T=300\text{K}$

$$V_{\text{rms}} = (5.4 \pm 0.1)\text{mV}$$

$$\chi_{\text{red}}^2 = 1.04$$

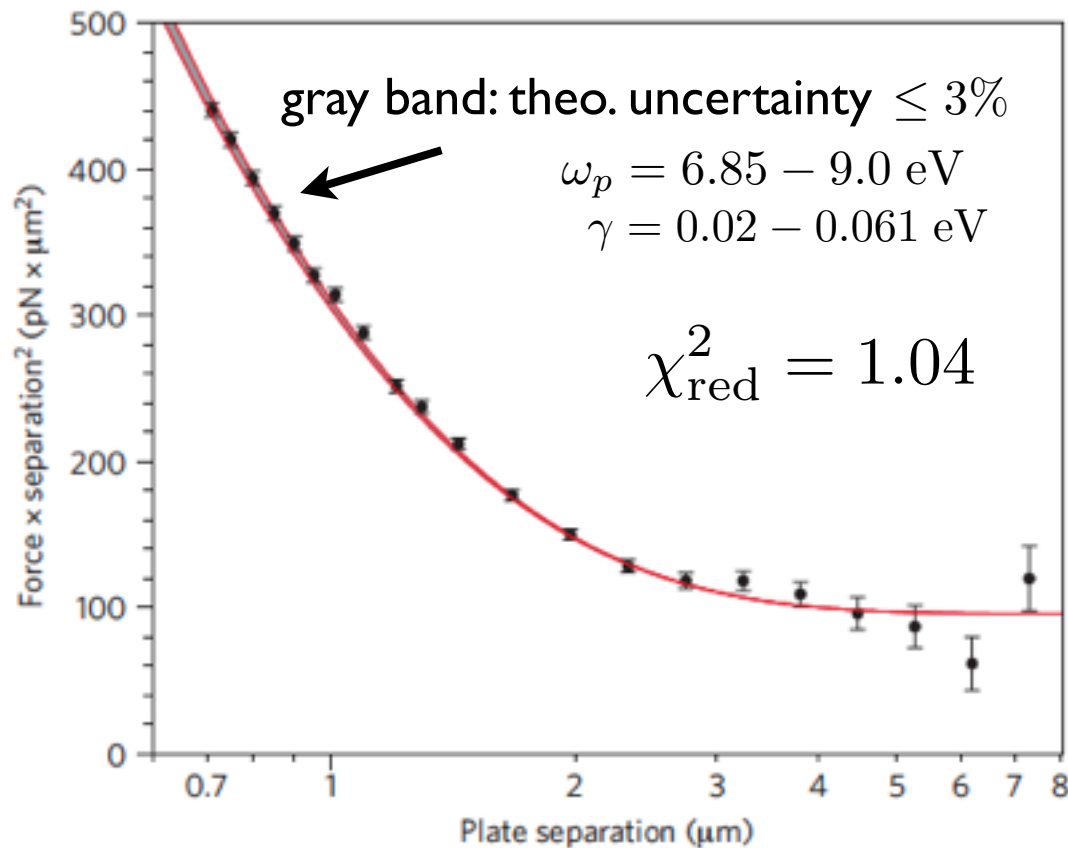
The other three models do not fit this description

Plasma $T=300\text{K}$: $\chi_{\text{red}}^2 = 32$

Drude $T=0\text{K}$: $\chi_{\text{red}}^2 = 23$

Plasma $T=0\text{K}$: $\chi_{\text{red}}^2 = 43$

The thermal Casimir force



Thermal Casimir force


$$d^2 F_{\text{Drude}}^{(T)}(d) \rightarrow \frac{\xi(3) R k_B T}{8} = 97 \text{ pN } \mu\text{m}^2$$

(large separations)

Remarks on the Au experiment

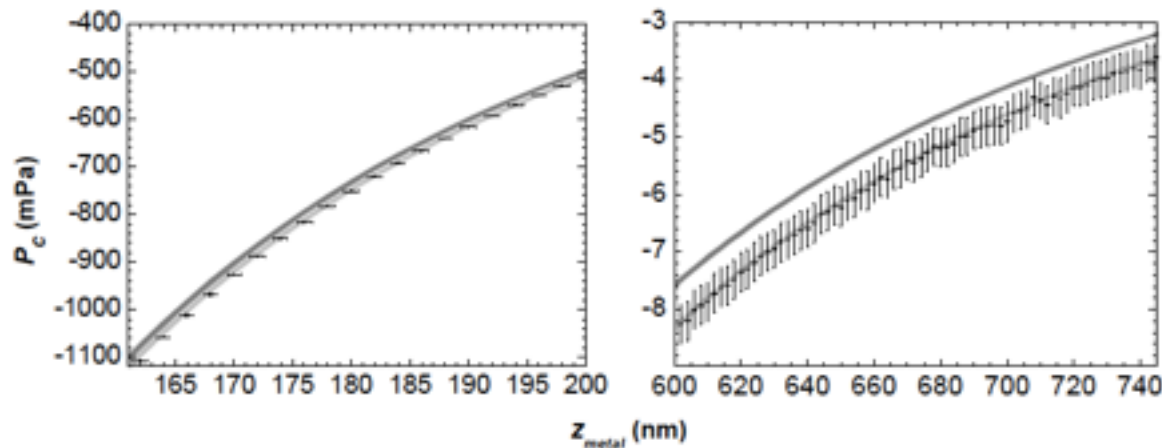
 Observation of the thermal Casimir force.

- modeled patch contribution
- modeled Casimir contribution

 Our measurement and analysis indicate that the Drude model to describe Casimir interactions in metallic plates is correct.

Global remarks

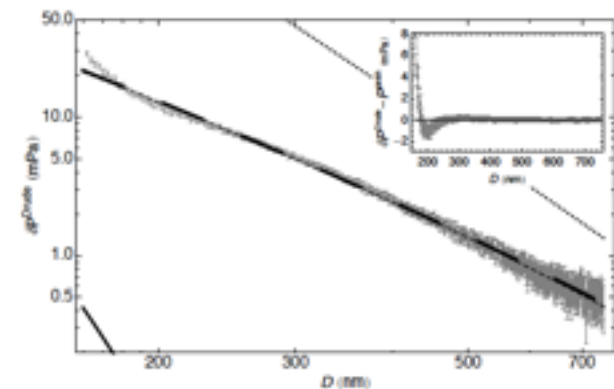
- Other experiments seem to be compatible with plasma model



E.g.: Decca group

- Better modeling of patches is needed

PRA **85**, 012504 (2012) [Ryan Behunin, next week]



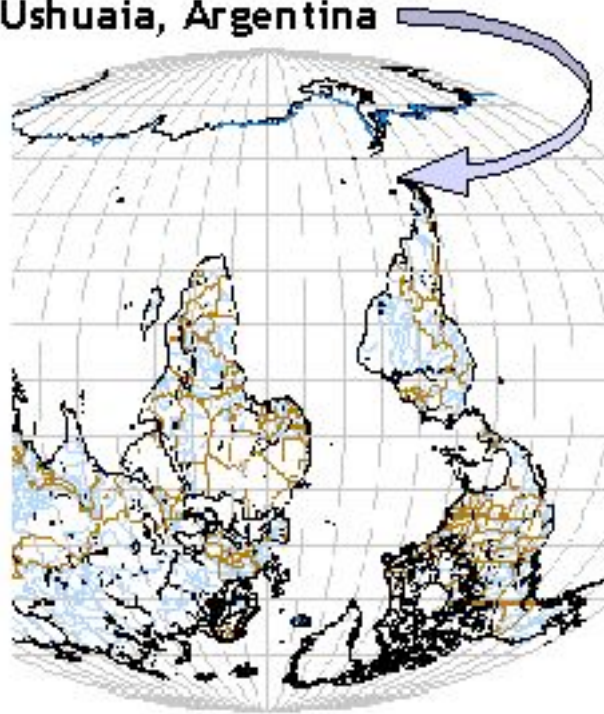
- Measurements of patches are needed

NSF Pan American Advanced Study Institute (PASI)
School/Workshop in October 2012 on

Frontiers in Casimir Physics

LOCATION:

Ushuaia, Argentina



Organizers: R. Decca, DD, R. Esquivel-Sirvent, P. Maia Neto, D. Mazzitelli, and H. Pastoriza